

# ClimODE: Climate and Weather Forecasting with Physics-Informed Neural ODEs

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## Objective

Model and forecast weather and climate:

$$\mathbf{u}(\mathbf{x}, t) = \begin{pmatrix} u_1(\mathbf{x}, t) \\ \vdots \\ u_K(\mathbf{x}, t) \end{pmatrix} \begin{matrix} \text{Temperature} \\ \vdots \\ \text{Precipitation} \end{matrix} \quad \mathbf{u}(\mathbf{x}, t) = \mathbf{u}(\mathbf{x}, 0) + \int_0^t \underbrace{\dot{\mathbf{u}}(\mathbf{x}, t)}_{\text{unknown}} dt$$

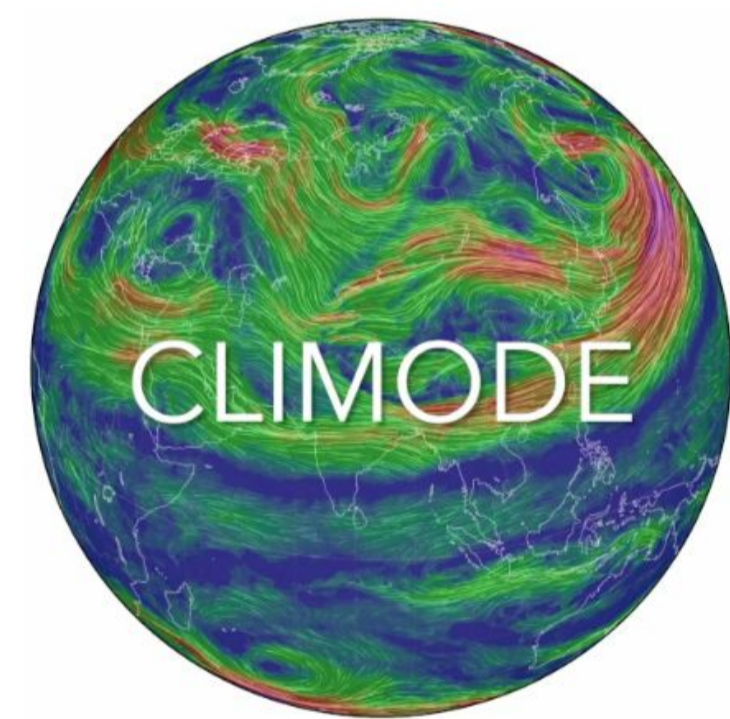
### Issues with black box modeling

- Black box methods based on Transformers, UNets, etc overlook the **physical dynamics (P)** and **continuous time (CT) nature** and are not **compact (C)**.

	P	CT	C
Transformer	✗	✗	✗
Neural PDE	✗	✓	✗
ClimODE	✓	✓	✓

### Contributions

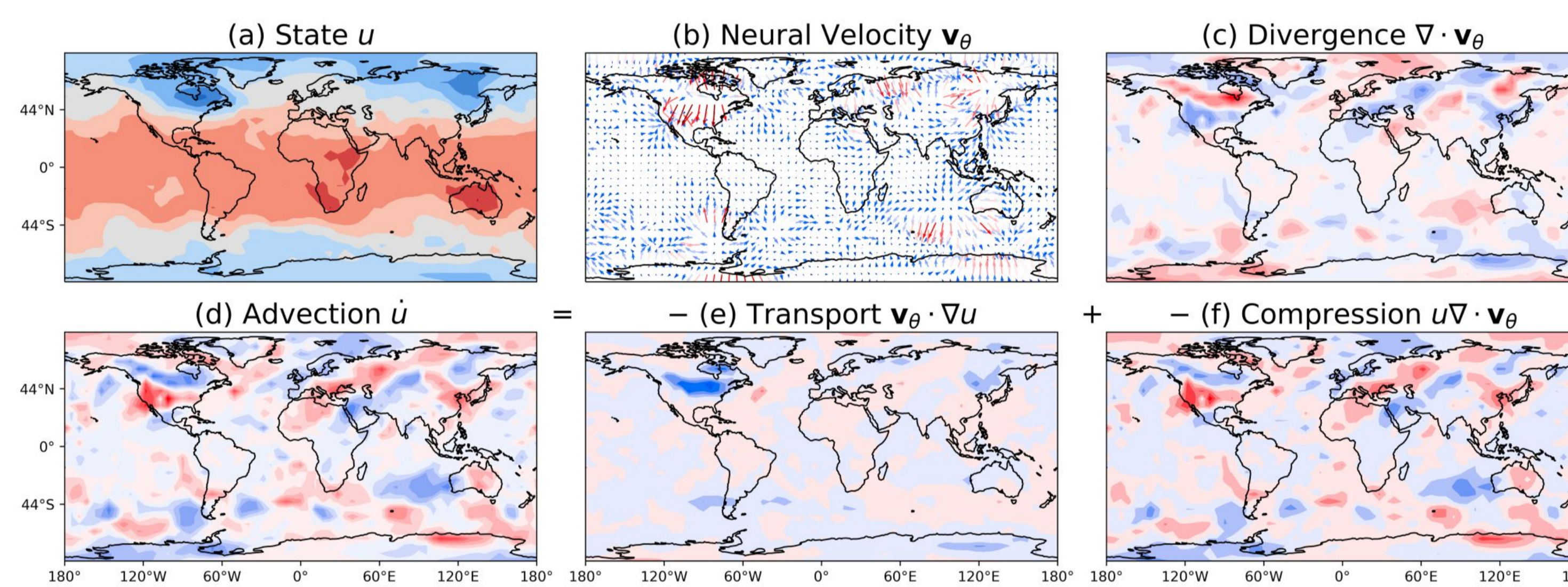
- Develop Neural ODEs/PDEs that
  - Respect **P**, are **CT** and **C**
  - Provide uncertainty estimates



## Neural Transport Model

We model weather/climate as a spatiotemporal process  $\mathbf{u}(\mathbf{x}, t) = (u_1(\mathbf{x}, t), \dots, u_K(\mathbf{x}, t))$  of  $K$  quantities as an advection PDE:

$$\dot{u}_k(\mathbf{x}, t) = -\mathbf{v}_k^T(\mathbf{x}, t) \nabla u_k(\mathbf{x}, t) - u_k(\mathbf{x}, t) \text{tr}(\nabla \mathbf{v}_k(\mathbf{x}, t))$$



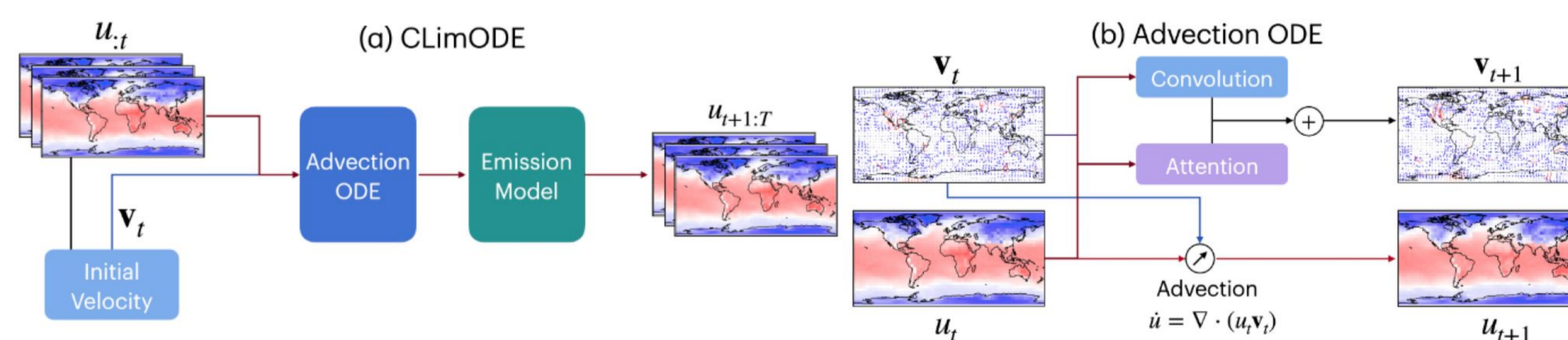
### Flow velocity $\mathbf{v}$

We model change in the velocity with a neural flow, as a function of observed values  $\mathbf{u}(t)$ ,  $\nabla \mathbf{u}$ ,  $\mathbf{v}(t)$  and spatio-temporal embeddings  $\Psi$

$$\dot{\mathbf{v}}_k(\mathbf{x}, t) = f_\theta(\mathbf{u}(t), \nabla \mathbf{u}(t), \mathbf{v}(t), \Psi)$$

To capture **local** and **global** effects, we propose a hybrid network

$$f_\theta(\mathbf{u}(t), \nabla \mathbf{u}(t), \mathbf{v}(t), \Psi) = f_{\text{conv}}(\mathbf{u}(t), \nabla \mathbf{u}(t), \mathbf{v}(t), \Psi) + \gamma f_{\text{att}}(\mathbf{u}(t), \nabla \mathbf{u}(t), \mathbf{v}(t), \Psi)$$



## Training Objective

Optimise the log likelihood over the observations,

$$\log p(\mathbf{y} | \theta, \phi) \propto \sum_{i=1}^N \log \mathcal{N}(\mathbf{y}_i | \mathbf{u}_\theta(t_i) + \boldsymbol{\mu}_\phi(t_i), \text{diag}(\boldsymbol{\sigma}_\phi(t_i)))$$

- Solve  $\mathbf{u}(t)$  forward
- Evaluate likelihood
- Backpropagate wrt  $\theta, \phi$

$$\begin{bmatrix} \mathbf{u}(t) \\ \mathbf{v}(t) \end{bmatrix} = \begin{bmatrix} \mathbf{u}(t_0) \\ \mathbf{v}(t_0) \end{bmatrix} + \int_{t_0}^t \begin{bmatrix} \dot{\mathbf{u}}(\tau) \\ \dot{\mathbf{v}}(\tau) \end{bmatrix} d\tau$$

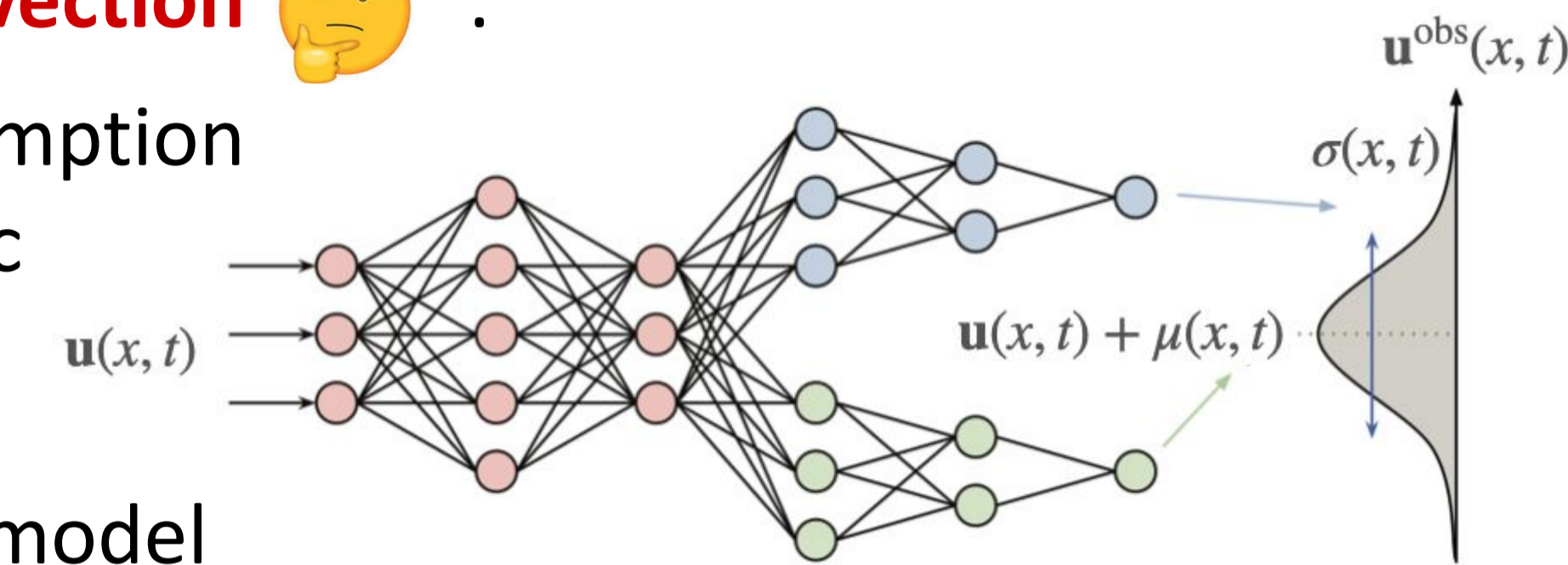
## Sources and Uncertainty

**Limitation of vanilla advection** 🤔 :

- Closed system assumption
- ODE is deterministic

**Idea** 💡 :

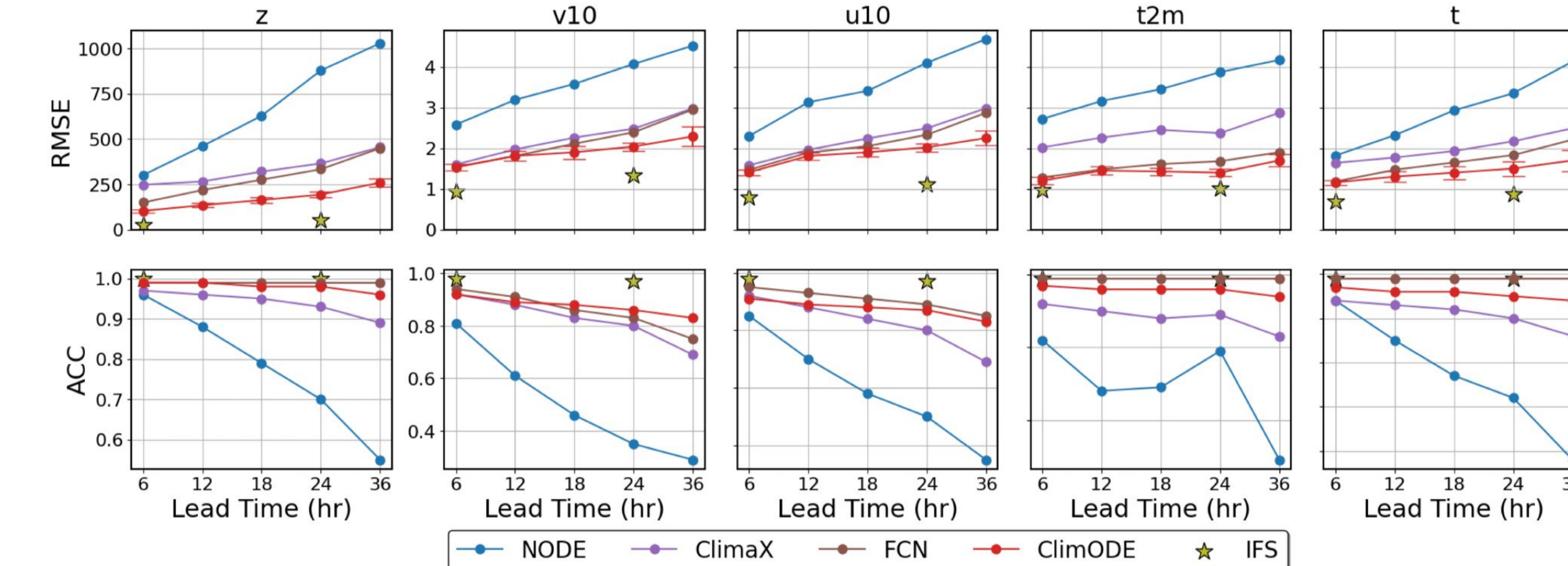
- Gaussian emission model
- Model predicts mean forecasts and uncertainty



$$u_k^{\text{obs}}(\mathbf{x}, t) \sim \mathcal{N}(u_k(\mathbf{x}, t) + \mu_k(\mathbf{x}, t), \sigma_k^2(\mathbf{x}, t)), \quad \mu_k(\mathbf{x}, t), \sigma_k(\mathbf{x}, t) = g_\phi(\mathbf{u}(\mathbf{x}, t), \Psi)$$

## Experiments

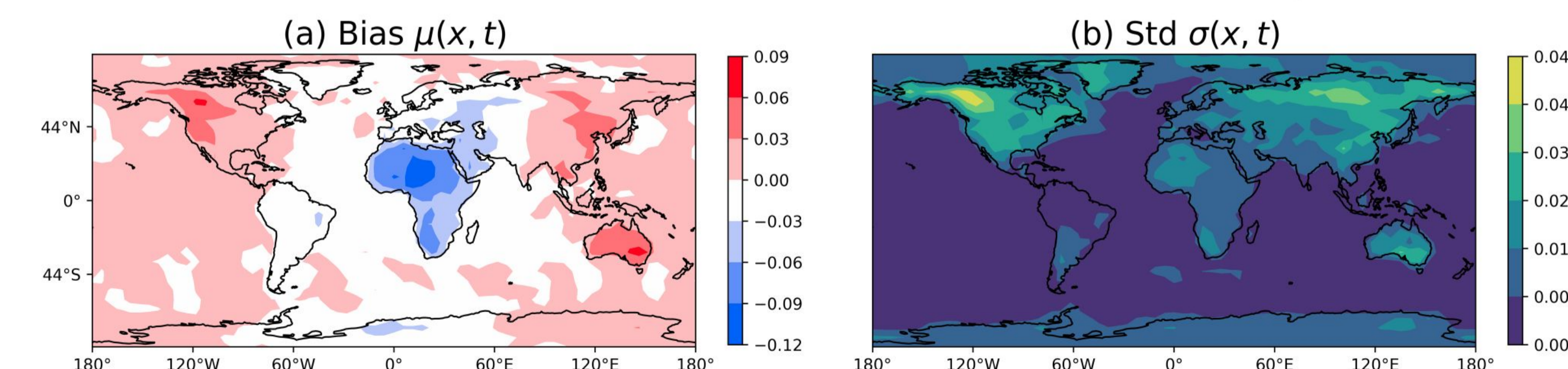
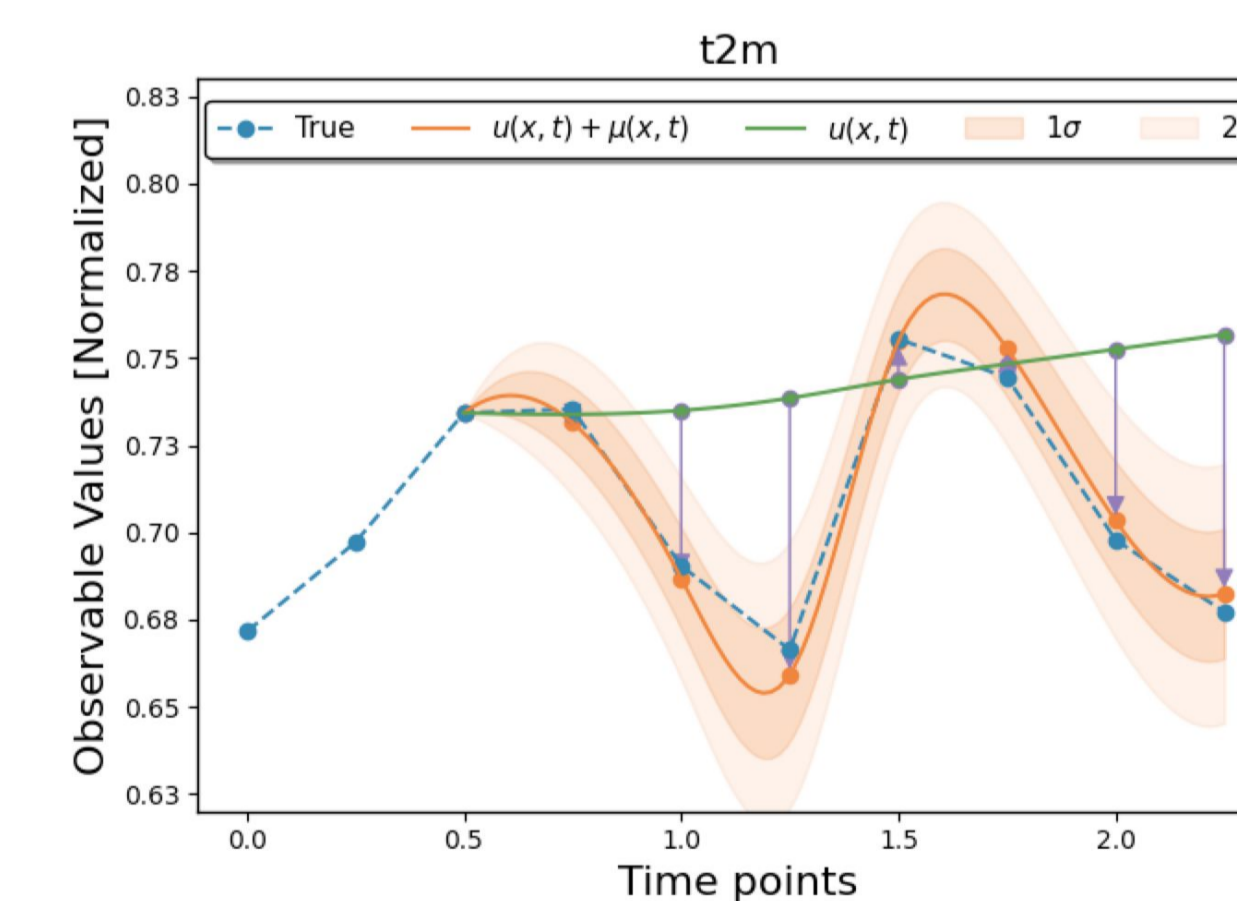
### Global Forecasting



### Interpretability

**Bias:** Explains day-night cycle

**Uncertainty:** Highest on land and in north according to diurnal cycle



## Advection PDE

- Movement of quantities  $u$  (scalar field) due to velocity  $\mathbf{v}$  (vector field)
- Value-conserving dynamics

$$\frac{\partial u}{\partial t} + \mathbf{v} \cdot \nabla u + u \nabla \cdot \mathbf{v} = 0$$

Time evolution  $\dot{u}$

## References

- Climax: A foundational model for weather and climate, ICML 2023
- FourCastNet: A Global Data-driven High resolution weather model, PASC 2023
- Learning skillful medium-range global weather forecasting, Science, 2023
- ECMWF, IFS Documentation CY48R1
- Neural Ordinary Differential Equations, NeurIPS 2018